**MODULE 1**

**UNIT 1 MATRICES**

**1.1 Definition and Examples**

A matrix is a rectangular array of numbers. In the array, there are vertical and horizontal elements. An matrix is a matrix with rows and columns. A matrix can be denoted by , . The elements of a matrix , can be denoted by

For example, in the matrix , =8.

**1.2 Different types of matrices**

* Row matrix: A matrix which has exactly one row is called a row matrix e.g .
* Column matrix: A matrix which has exactly one column is called a column matrix e.g .
* Square matrix: A matrix in which the number of rows is equal to the number of columns is called a square matrix. E.g is a 2 x 2 square matrix. is a 3 x 3 square matrix.
* Null or zero matrix: A matrix each of whose elements is zero is called a Null Matrix or Zero Matrix. E.g is a 2 x 3 Null Matrix.
* Diagonal Matrix: The elements where in a square matrix , are called the diagonal elements. For example in , the diagonal elements are ,

A square matrix whose every element other than diagonal elements is zero is called a Diagonal Matrix.

For example, is a Diagonal Matrix.

Note that, the diagonal elements in a diagonal matrix may also be zero. For example: and are also diagonal matrices.

* Scalar Matrix: A diagonal matrix whose diagonal elements are equal is called a Scalar matrix. For example. , , are scalar matrices.
* Identity Matrix: A diagonal matrix whose diagonal elements are all equal to 1, is called Identity Matrix or Unit Matrix. For example, is an identity matrix.
* Triangular Matrix: A square matrix whose elements =0 whenever is called a Lower Triangular Matrix. Similarly, a square matrix whose elements whenever is called Upper Triangular Matrix.

For example, , are lower triangular matrices and are upper triangular matrices.

* Singular matrix: This is a square matrix whose determinant is zero. It is not an invertible matrix.
* Symmetric matrix: A square matrix is called a symmetric matrix if for all values of and or for a matrix
* Skew symmetric matrix: A square matrix is called a skew symmetric matrix if for all values of and or for a matrix Also, is said to be a skew symmetric matrix if all the diagonal elements are zero.

**MODULE 2**

**UNIT 1. ALGEBRA OF MATRICES.**

The major arithmetic operations performed on matrices are addition, subtraction, scalar multiplication ,multiplication and inverse( division).

**1.1 Addition of matrices.**

If and are two matrices of the same order, then the addition of and is defined to be the matrix obtained by adding the corresponding elements of and For example, , Then = .

**1.2 Subtraction of matrices.**

If and are two matrices of the same order, then the subtraction of from is defined to be the matrix obtained by subtracting the corresponding elements of from For example, , =.

**1.3 Multiplication by a Scalar.**

If is any complex number and a given matrix, then is the matrix obtained from by multiplying each element of by The number is called a scalar. For example, if and then .

Exercise:

1. Show that
2. Show that

**1.4 Multiplication of matrices.**

The product of two matrices A and is defined only when the two matrices are conformable i.e when the number of columns of equals the number of rows of We shall consider the following example.

Let and let . Then .

The method can be similarly used for other matrices whose orders are conformable.

**1.5 Properties of matrices.**

* Matrix addition is commutative i.e Matrix addition is associative i.e
* Matrix subtraction is neither commutative nor associative i.e
* Matrix product is not commutative but associative. i.e
* The transpose of a matrix say denoted by or is the matrix obtained by interchanging its rows and columns. For example, if then

**Exercise.**

Prove or disprove the following statements.

* Elementary operations of matrices. Consider the matrices

Matrix is obtained from by the interchange of first and second row.

Consider .

Matrix is obtained from by multiplying first row by 3.

Consider .

* Matrix is obtained from by multiplying first row of by 3 and adding it to second row.
* Such operations on rows of a matrix as described above are called Elementary row operations.

We similarly have Elementary column operations.

**MODULE 3**

**UNIT1. DETERMINANT OF A MATRIX.**

Let be a matrix. The determinant of is denoted by .

Determinant of order one. Let be a square matrix of order one. Then det =.

Determinant of order two. Let be a square matrix of order two. Then . For example, if , then

Determinant of order three. Let be a 3 matrix. Then the det .

Let . To find determinant of , we take the following steps.

We remove and find the determinant of the remaining matrix: =

We remove and find the determinant of the remaining matrix :

We remove and find the determinant of the remaining matrix:

By using det , we have

**UNIT 2 INVERSE OF A MATRIX.**

If and are two square matrices of the same other, such that , then is called the inverse of i.e and is the inverse of i.e It should be noted that for a square matrix to possess an inverse , it must be a singular matrix i.e

For every non singular matrix where is the transpose of a Cofactor of

We shall now illustrate the Adjoint of with an example.

Let . The Co-factor of elements of various rows of are found as follows.

We find the determinant of each of the elements of matrix as follows.

Note that the determinant of each element of matrix is called the minor of the element.

= Co-factor of matrix

Now to find the Adjoint of we find the transpose of the co-factor obtained above. Thus we have Suppose we are to find the inverse of the above matrix A, we now find and use the above formular.

Using we have

**MODULE 4**

**UNIT 1 SOLUTION OF LINEAR SYSTEM OF EQUATIONS.**

In this section we shall consider two methods of solving linear system of equations. These methods are (i) Determinant method (Cramer’s Rule) (ii) Inverse method. Other methods of solution are: Row reduction method and Gauss Elimination method.

The following is an example of a linear system of equations.

where are constants and are unknown variables.

This sytem of linear equations could be written in a matrix form as

* **Solution by Cramer’s Rule.**

Example: Solve the following system of equations using Cramer’s rule.

.

Solution: We write in matrix form as

Let =5(48+2)+7(-36+3)+1(12+24)=55.

Interchanging the first column in the matrix with gives the new matrix . Now let =11(48+2)+7(-90+7)+1(30+56)=55.

Interchanging the second column in the matrix with gives =5(-90+7)-11(-36+3)+1(42-45)=-55.

Interchanging the third column in the matrix with gives =5(-56-30)+7(42-45)+11(12+24)=-55.

Now

**Assignment** : Solve the following equations using Cramer’s rule.

. . .

. . .

* **Solution by Inverse method.**

We shall use the inverse method to solve the previous example in order to verify the workability of the two methods of solution being considered.

Example: Solve the following system of equations using inverse method.

.

The system of equations can be written in matrix form as

Now the Cofactors of coefficient matrix are found as: =48+2=50. =-(-36+3)=33. =12+24=36. = -(42-2)= -40. =-30-3=-33. =-(10+21)=-31. =(7+8)=15. =-(-5-6)=11. =(-40+42)=2.

Matrix of cofactors= . Adjoint matrix= . Let matrix . =. From the linear form ==.